



**Supporting the continuation of teaching STEM subjects
during the COVID-19 Pandemic through project-based
online practices**

Mathematics of soap bubbles

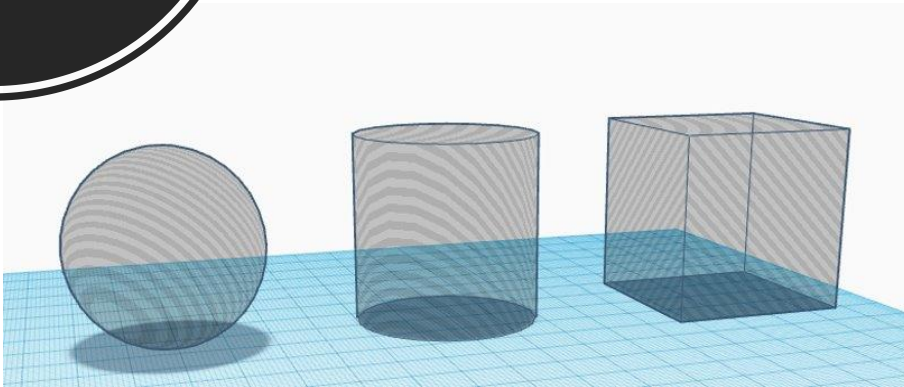
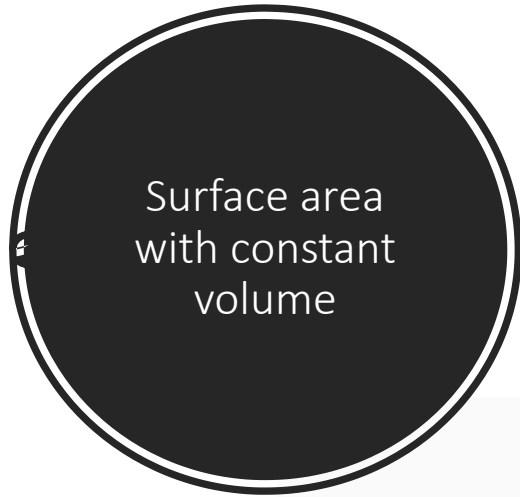
Mateusz Kośnik

Queen Hedwig's 10th Secondary School in Warsaw

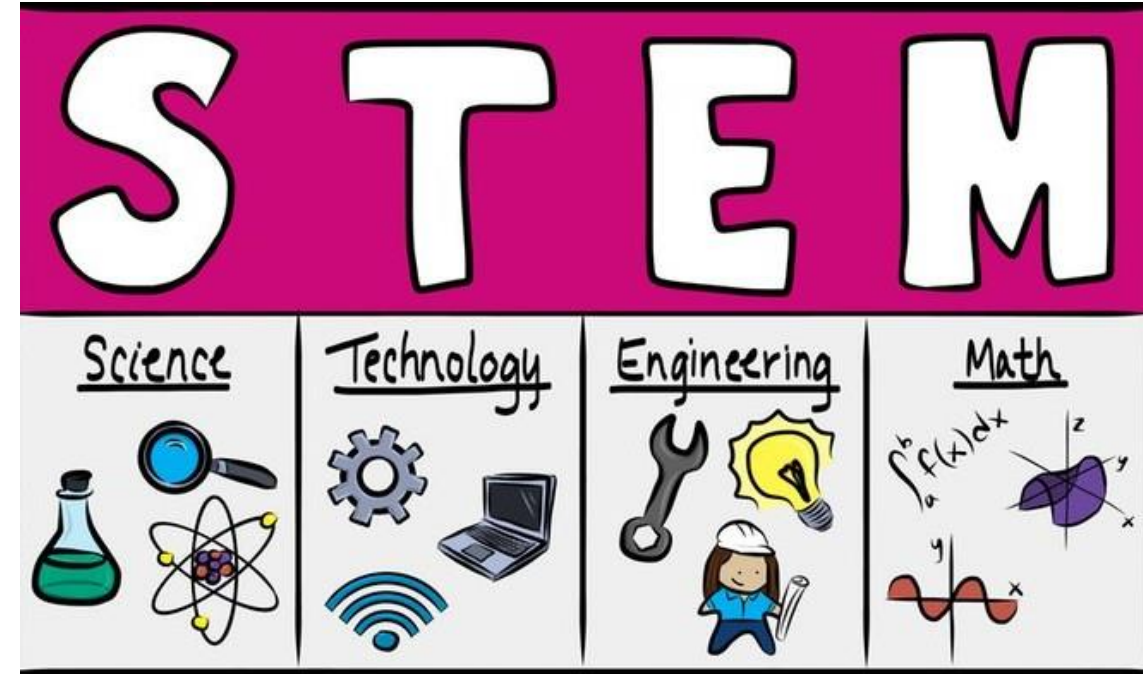
Presentation plan:

1. Aims of the project
2. Open Educational Resources use
3. Project implementation

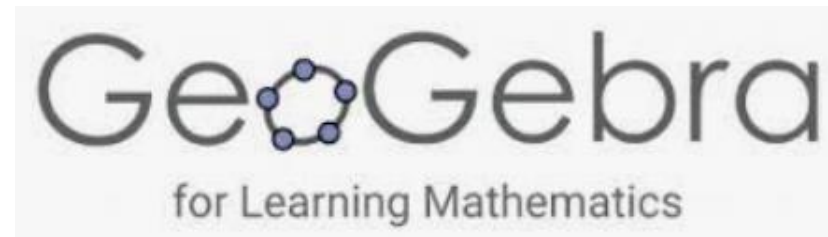
Aims of the project



Which body would have the smallest surface area given a constant volume?



Open Educational Resources

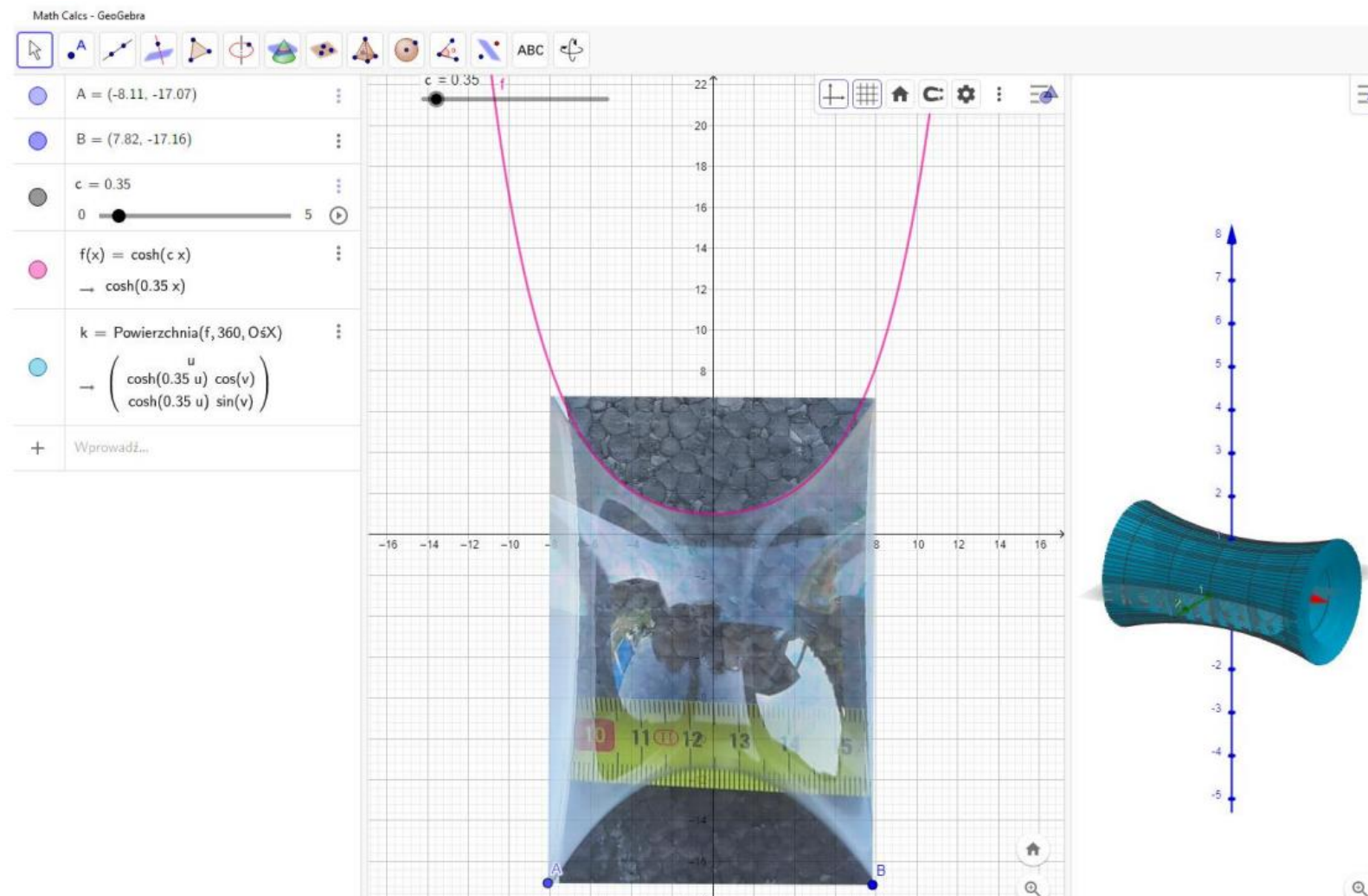


Project implementation – experiment preparation



- 1) soapy water
- 2) rings made from cream/yogurt cups
- 3) tape measure, ruler, graph paper
- 4) background adjustment

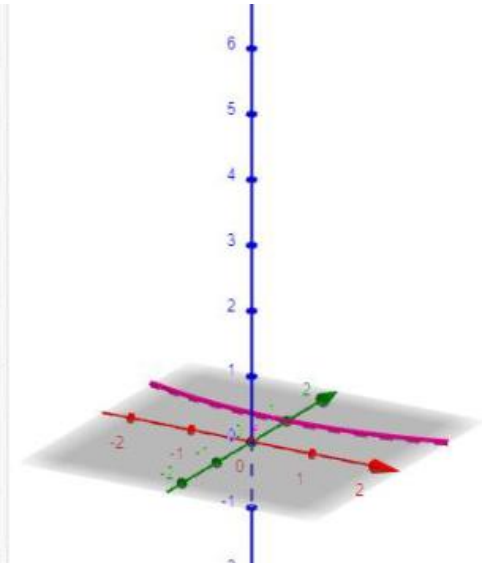
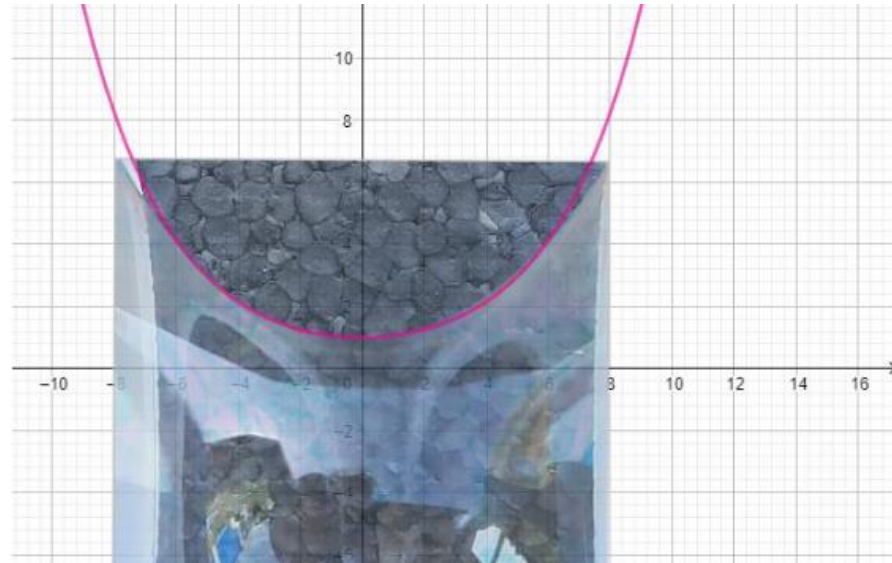
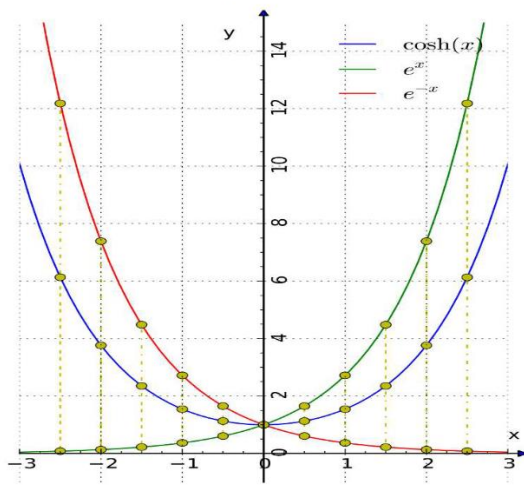
Project implementation – curve matching in GeoGebra



Project implementation – catenoid's surface area calculation

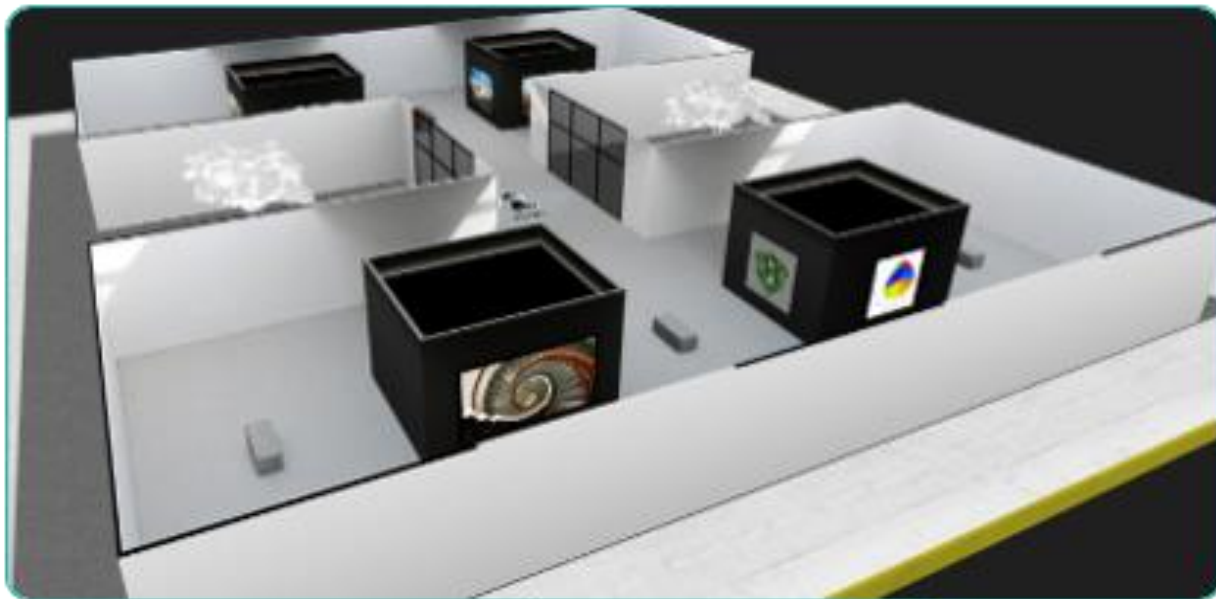
$$S = \frac{\pi}{c^2} \left(2c \sinh ca + \ln \left| \frac{\sqrt{1 + c^2 \sinh^2 ca} + c \sinh ca}{\sqrt{1 + c^2 \sinh^2 ca} - c \sinh ca} \right| \right)$$

$$y = f(x) = \cosh x = \frac{e^x + e^{-x}}{2}$$



surface area of a catenoid is smaller than that of a cylinder

Project implementation – minimum surface in everyday life



Project implementation – students' output

**„KSZTAŁT WODY”
POWIERZCHNIE MINIMALNE**

X LO
OGÓLNOKSZTAŁCĄCE
IM. KRÓLOWEJ JADWIGI

Erasmus+

Galeria Artsteps

Geogebra - projekt katenoidy

Ostatecznie pole powierzchni katenoidy obliczamy stosując poniższy wzór

$$S = \frac{\pi}{c^2} \left(2c \sinh ca + \ln \frac{\sqrt{1+c^2 \sinh^2 ca} + c \sinh ca}{\sqrt{1+c^2 \sinh^2 ca} - c \sinh ca} \right)$$

Nasze $c=1.15$ i $a=1$ dlatego pole naszej katenoidy to
w przybliżeniu 17,45069

Pole boczne walca $P_b = 2\pi rh$ **= 6,94988 π**

Erasmus+ **KSZTAŁT WODY
POWIERZCHNIE MINIMALNE**

**TWORZENIE
KATENOIDY**

**PROJEKT KATENOIDY
W PROFESJONALNYM
PROGRAMIE**

$$S = \frac{\pi}{c^2} \left(2c \sinh ca + \ln \frac{\sqrt{1+c^2 \sinh^2 ca} + c \sinh ca}{\sqrt{1+c^2 \sinh^2 ca} - c \sinh ca} \right)$$

tak obliczyliśmy pole powierzchni katenoidy
Do porównania pole
powierzchni bocznej walca
 $P_w = 2\pi \cosh(0,35a)2a$
 $P_w = 312,3154$

**GALERIA
W ARTSTEPS**

Stanisław, Bartek, Weronika, Tsimafei
Źródła: Polskie Towarzystwo Matematyczne

Projekt Erasmus
"Kształt wody" -
powierzchnie minimalne

X LO. im. Królowej Jadwigi
Partnerzy :

Erasmus

Doświadczenie

- Uzyskanie katenoidy za pomocą płynu do baniek, cyny i dwóch pokrywek od jogurtu.

Wpasowanie krzywej
w geogebrze
oraz wyniki obliczeń

Screenshots z artsteps
Galeria Artsteps

Przykłady powierzchni minimalnej

$$S = \frac{\pi}{c^2} \left(2c \sinh ca + \ln \frac{\sqrt{1+c^2 \sinh^2 ca} + c \sinh ca}{\sqrt{1+c^2 \sinh^2 ca} - c \sinh ca} \right)$$

$$P_b = 2\pi rh$$

$a=1$
 $c=0.7$

- P_b walca = 15,77292
- S katenoidy = 13,33294

Brali udział: K.K, M.M, M.D, A.K

The BeReady partnership



Get in touch



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